FLUID MECHANICS

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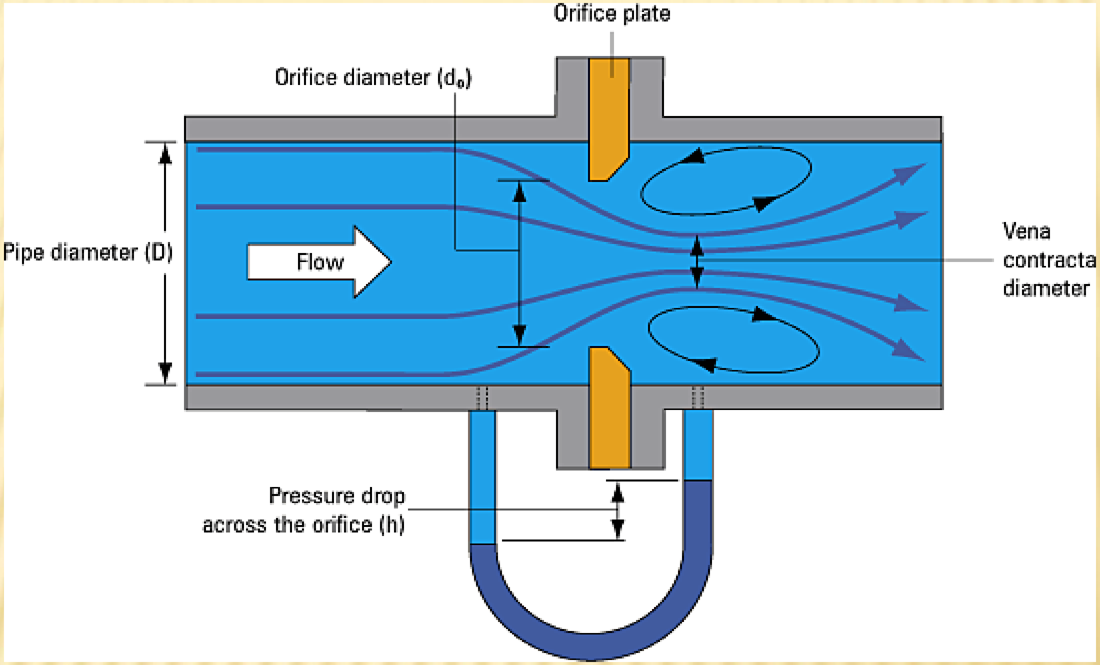
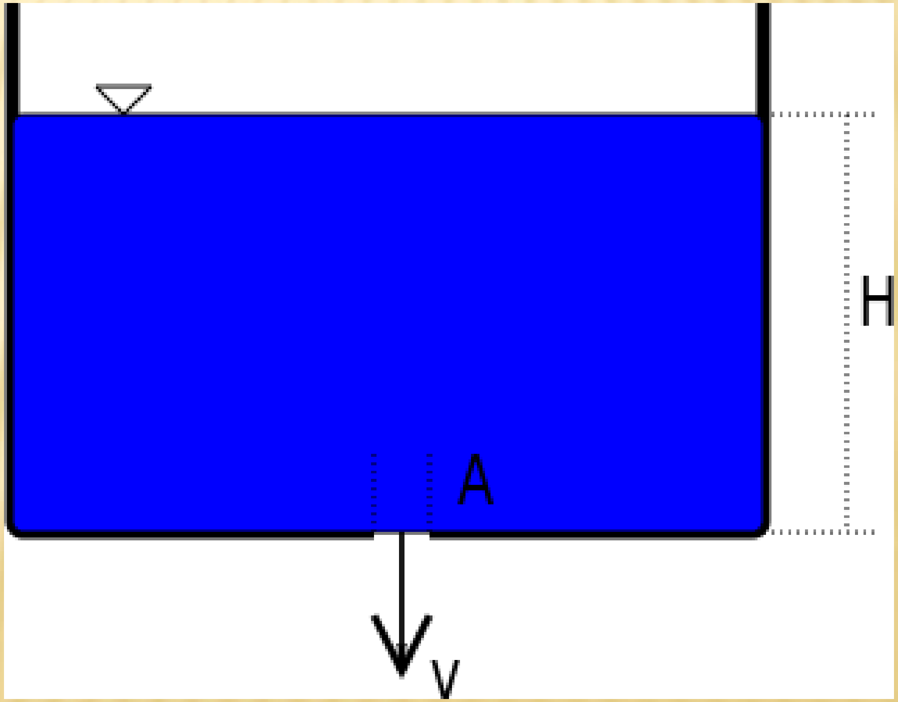
FLOW MEASUREMENTS



FLOW THROUGH ORIFICE

- What is a Orifice?

Orifice is a small opening of any cross-section(circular, triangular, rectangular etc.) on the side or bottom of the tank through which fluid is flowing.



FLOW THROUGH ORIFICE

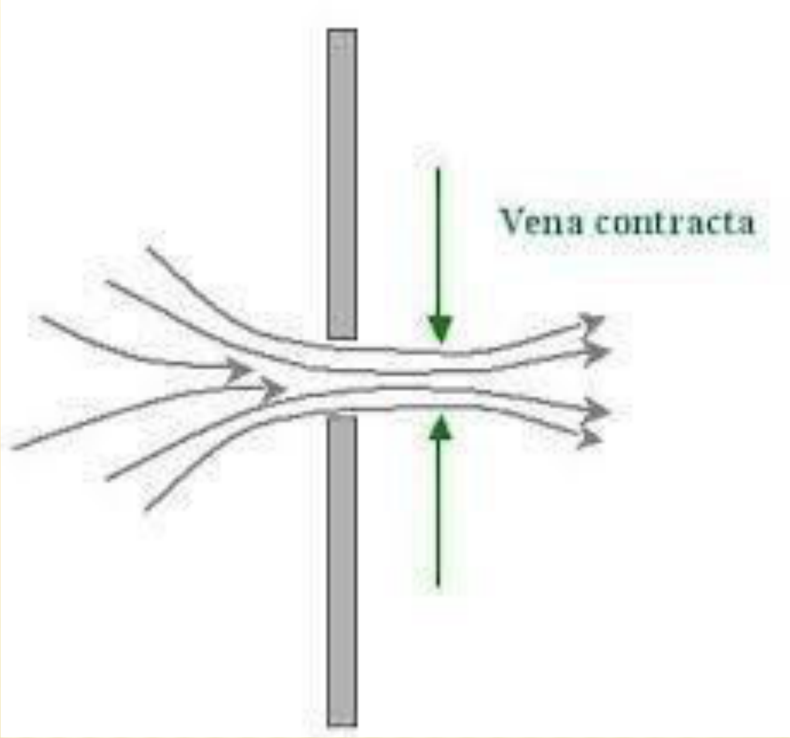
What is a mouthpiece?

A mouthpiece is a short pipe of length two or three times its diameter fitted in a tank/vessel containing fluid.

Both of them are used for measuring the rate of flow of fluid.

VENA-CONTRACTA

- The liquid coming out from an orifice forms a jet of liquid whose cross-sectional area is less than that of orifice.
- The area of jet of fluid goes on decreasing and at a section ,it becomes minimum.
- This section is approximately at a distance of half of diameter of orifice.
- Beyond this section the jet diverges and is attracted to downward direction by gravity.



This section is called Vena-contracta.

DERIVATION OF THEORETICAL VELOCITY AT VENA-CONTRACTA

Consider two points 1 and 2 as shown in figure.

Point 1 is inside the tank and point 2 is at vena-contracta.

Let the flow is steady and at constant head H.

Applying Bernoullis equation at point 1 and 2,

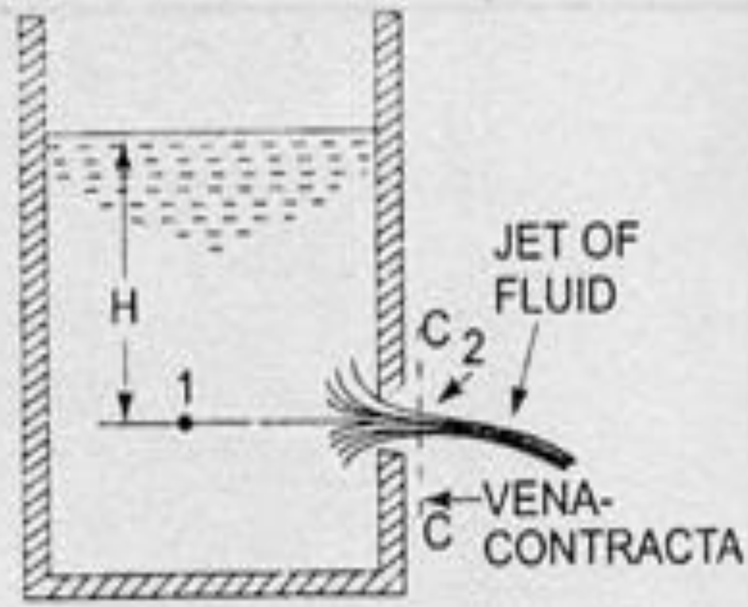


Fig. 7.1 *Tank with an orifice.*

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

But

$$z_1 = z_2$$

DERIVATION OF THEORETICAL VELOCITY AT VENA CONTRACTA

$\therefore \quad \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$

Now $\quad \frac{p_1}{\rho g} = H$

$\frac{p_2}{\rho g} = 0$ (atmospheric pressure)

v_1 is very small in comparison to v_2 as area of tank is very large as compared to the area of the jet of liquid.

$\therefore \quad H + 0 = 0 + \frac{v_2^2}{2g}$

$\therefore \quad v_2 = \sqrt{2gH} \quad$ ...(7.1)

This is theoretical velocity. Actual velocity will be less than this value.

HYDRAULIC COEFFICIENTS

THE HYDRAULIC CO-EFFICIENTS ARE 1.COEFFICIENT OF VELOCITY, CV 2.COEFFICIENT OF CONTRACTION, CC 3.COEFFICIENT OF DISCHARGE, CD

• **Coefficient of Velocity(Cv)**

It is defined as the ratio between the actual velocity of a jet of liquid at vena-contracta and the theoretical velocity of the jet.

The value of Cv varies from 0.95 to 0.99 for different orifices depending on their shape ,size and on the head under which flow takes place. Generally 0.98 is taken as its value for sharp edged orifice.

$$C_v = \frac{\text{Actual velocity of jet at vena-contracta}}{\text{Theoretical velocity}}$$

$$= \frac{V}{\sqrt{2gH}}, \text{ where } V = \text{actual velocity}, \sqrt{2gH} = \text{Theoretical velocity}$$

HYDRAULIC COEFFICIENTS

- **Coefficient of Contraction(Cc)**

It is defined as the ratio of the area of the jet at vena-contracta to the area of the total opening. Its value varies from 0.61 to 0.69.Generally 0.64 can be taken as value of Cc.

$$C_c = \frac{\text{area of jet at vena-contracta}}{\text{area of orifice}}$$

HYDRAULIC COEFFICIENTS

- **Coefficient of discharge(Cd)**

It is defined as the ratio of actual discharge to the theoretical discharge from any opening. Its value lies between 0.61 to 0.65.Generally 0.62 is taken as its value.

$$C_d = \frac{Q}{Q_{th}} = \frac{\text{Actual velocity} \times \text{Actual area}}{\text{Theoretical velocity} \times \text{Theoretical area}}$$

$$= \frac{\text{Actual velocity}}{\text{Theoretical velocity}} \times \frac{\text{Actual area}}{\text{Theoretical area}}$$

$$C_d = C_v \times C_c$$

EXPERIMENTAL DETERMINATION OF HYDRAULIC COEFFICIENTS

Determination of Coefficient of discharge(Cd)

The water is allowed to flow through an orifice fitted to a tank under constant head H.

The water is collected in a measuring tank for known time t.The height of water in the measuring tank is noted down.

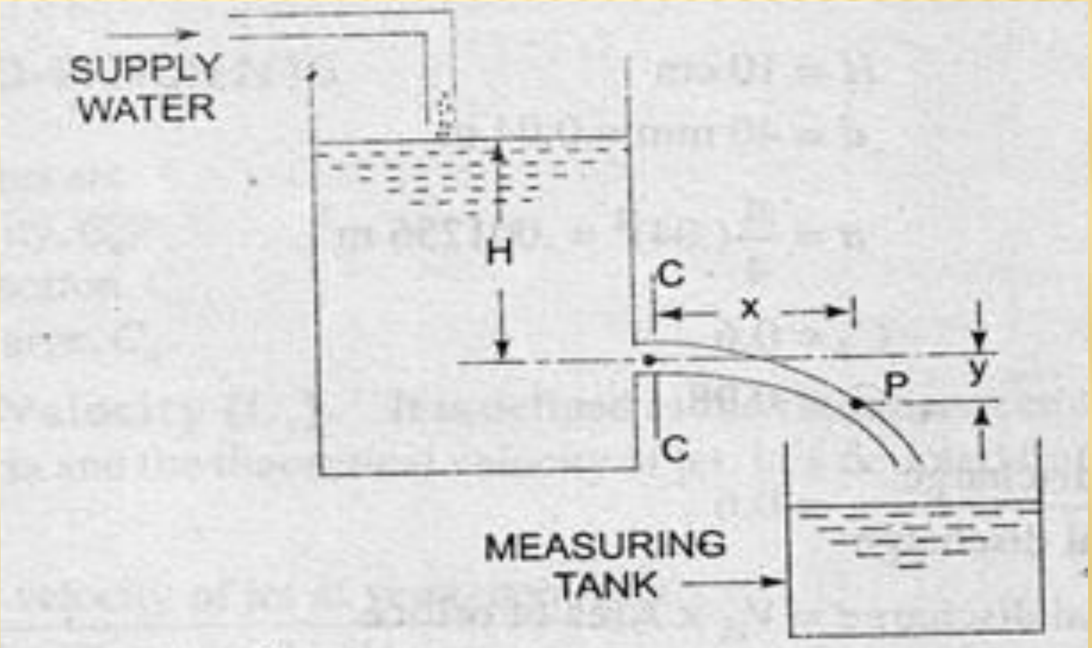


Fig. 7.2 *Value of C_d.*

EXPERIMENTAL DETERMINATION OF HYDRAULIC COEFFICIENTS

- Actual discharge through the orifice,

$$Q = \frac{\text{Area of measuring tank} \times \text{Height of water in measuring tank}}{\text{Time } (t)}$$

$$\text{theoretical discharge} = \text{area of orifice} \times \sqrt{2gH}$$

$$C_d = \frac{Q}{a \times \sqrt{2gH}}$$

EXPERIMENTAL DETERMINATION OF HYDRAULIC COEFFICIENTS

Determination of Coefficient of velocity(Cv)

Let C-C represent vena contracta of jet of water coming out from an orifice under constant head H. Consider a liquid particle which is at vena contracta at any time and takes position P along the jet in time t.

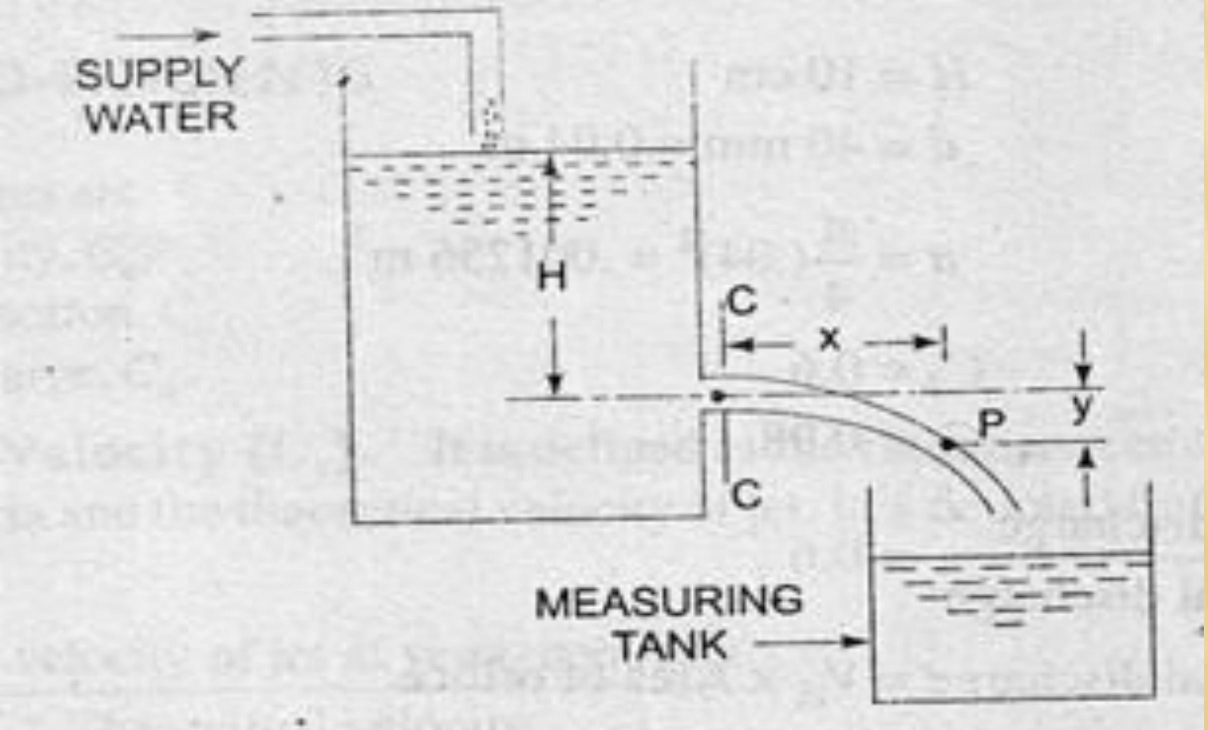


Fig. 7.2 *Value of C_d.*

EXPERIMENTAL DETERMINATION OF HYDRAULIC COEFFICIENTS

Let x = horizontal distance travelled by the particle in time 't'

y = vertical distance between P and C-C

V = actual velocity of jet at vena-contracta.

Then horizontal distance, $x = V \times t$ ...(i)

and vertical distance, $y = \frac{1}{2} gt^2$ ...(ii)

From equation (i), $t = \frac{x}{V}$

Substituting this value of 't' in (ii), we get

$$y = \frac{1}{2} g \times \frac{x^2}{V^2}$$

$$V^2 = \frac{gx^2}{2y}$$

$$\therefore \quad V = \sqrt{\frac{gx^2}{2y}}$$

EXPERIMENTAL DETERMINATION OF HYDRAULIC COEFFICIENTS

But theoretical velocity,

$$V_{th} = \sqrt{2gH}$$

$$\therefore \text{ Co-efficient of velocity, } C_v = \frac{V}{V_{th}} = \sqrt{\frac{gx^2}{2y}} \times \frac{1}{\sqrt{2gH}} = \sqrt{\frac{x^2}{4yH}}$$

$$= \frac{x}{\sqrt{4yH}}.$$

EXPERIMENTAL DETERMINATION OF HYDRAULIC COEFFICIENTS

Determination of Coefficient of contraction(Cc)

We can determine the coefficient of contraction from the following equations:

$$C_d = C_v \times C_c$$

$$C_c = \frac{C_d}{C_v}$$

CLASSIFICATION OF ORIFICE

Depending on size and head of liquid from center of orifice

- Small orifice: If the head of liquid from center of orifice >5 times depth of orifice
- Large orifice: If the head of liquid from center of orifice <5 times depth of orifice

Depending on their cross-sectional area

- Circular, Triangular, Rectangular and Square

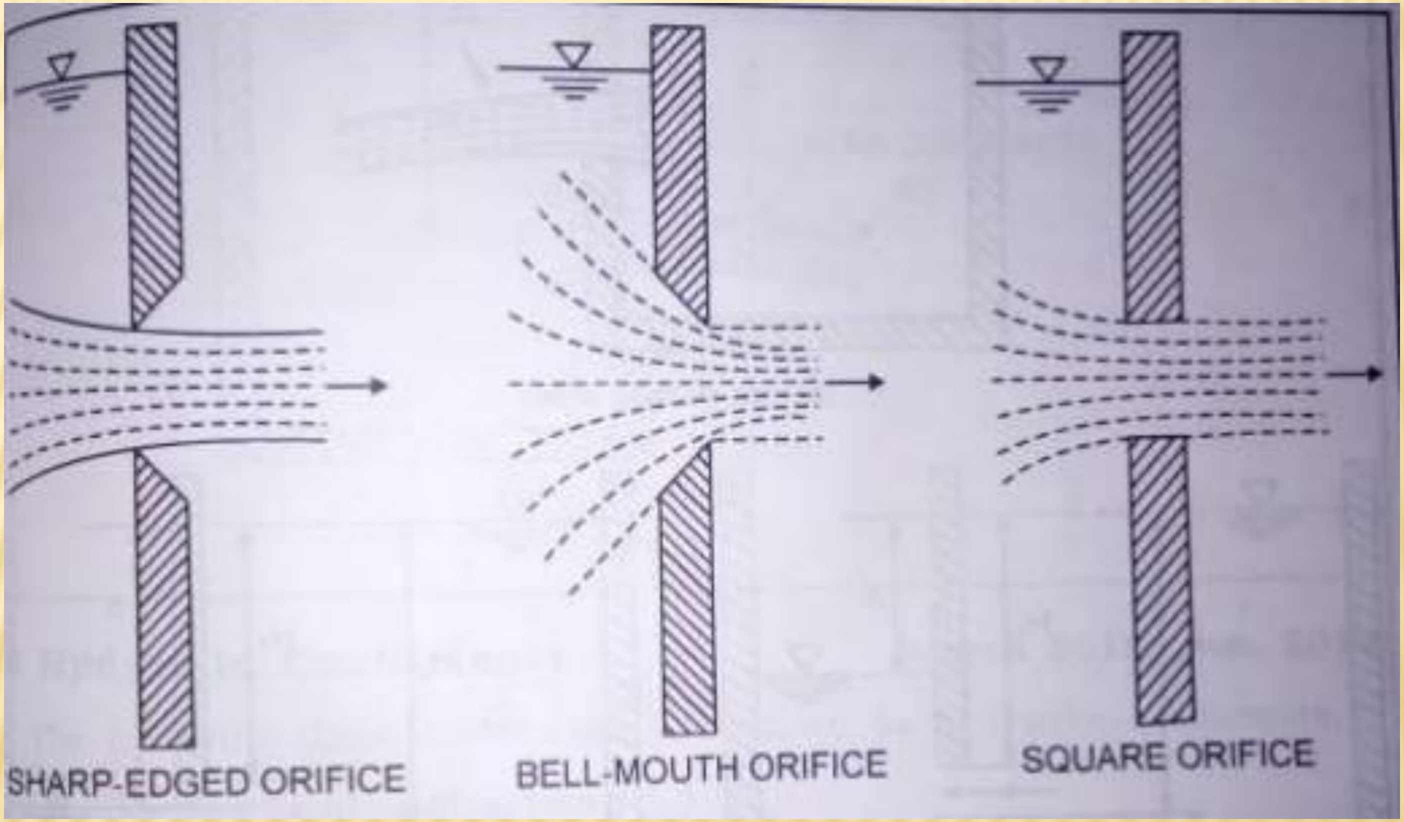
Depending on shape of u/s edge of orifice

- Sharp edged orifice
- Bell mouthed orifice

Depending on nature of discharge

- Free discharging orifice
- Fully Drowned/submerged orifice
- Partially Drowned/submerged orifice

DEPENDING ON SHAPE OF U/S EDGE OF ORIFICE



DEPENDING ON NATURE OF DISCHARGE

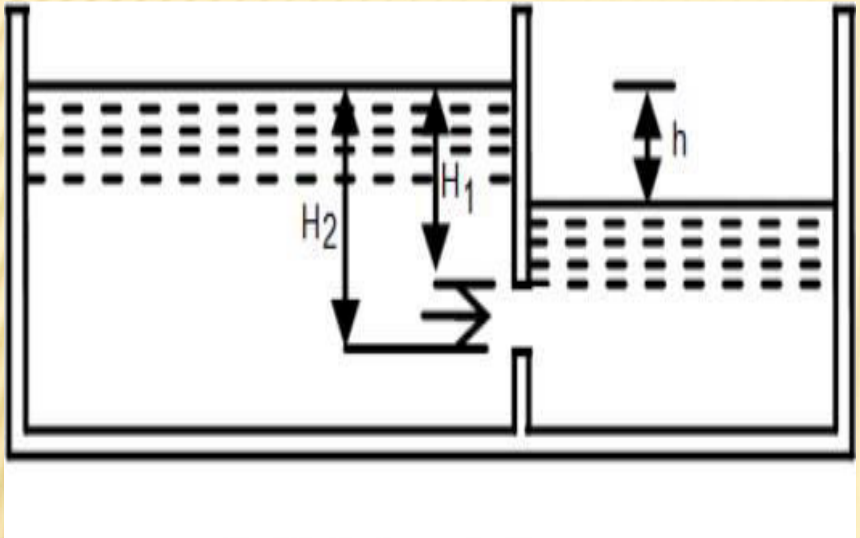


Fig: Wholly drowned orifice

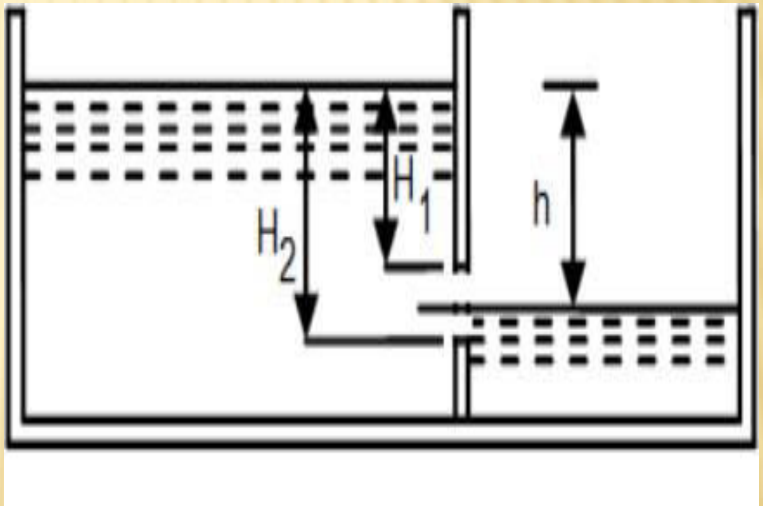


Fig: Partially drowned orifice

FLOW THROUGH SMALL ORIFICE

- In case of small orifice, the velocity in the entire cross-section of the jet is considered to be constant and discharge is calculated by

$$Q = C_d \times a \times \sqrt{2gh}.$$

FLOW THROUGH LARGE RECTANGULAR ORIFICE

- Consider a large rectangular orifice in one side of the tank discharging freely into atmoshphere under constant head H as shown in figure.

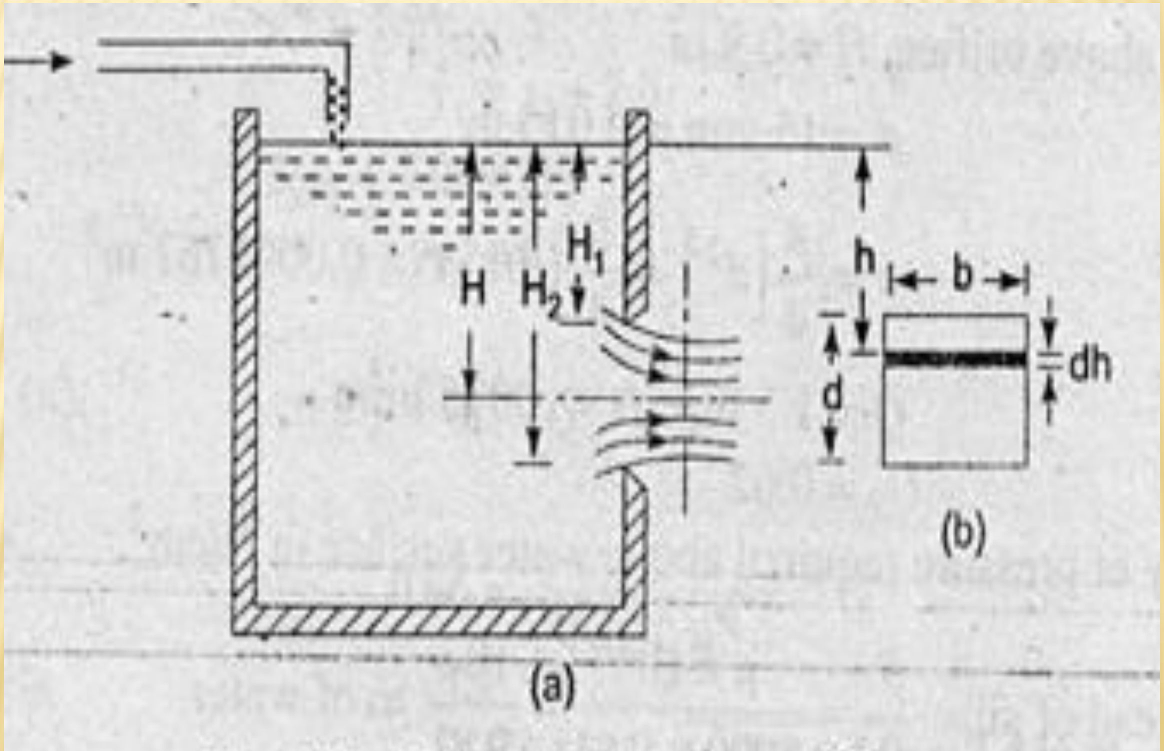


Fig. 7.7 *Large rectangular orifice.*

FLOW THROUGH LARGE RECTANGULAR ORIFICE

Let, H_1 =height of liquid above top edge of the orifice

H_2 =height of liquid above bottom edge of the orifice

b =breadth of orifice

d= depth of orifice= $H_2 - H_1$

C_d = Coefficient of discharge

Consider an elementary horizontal strip of depth dh at a depth h below the free surface of the liquid in the tank as shown in figure.

Area of strip=b*dh

and theoretical velocity of water through strip = $\sqrt{2gh}$.

$\therefore$ Discharge through elementary strip is given

$$dQ = C_d \times \text{Area of strip} \times \text{Velocity}$$

$$= C_d \times b \times dh \times \sqrt{2gh} = C_d b \times \sqrt{2gh}\, dh$$

By integrating the above equation between the limits H_1 and H_2, the total discharge through the whole orifice is obtained

FLOW THROUGH LARGE RECTANGULAR ORIFICE

$$Q = \int_{H_1}^{H_2} C_d \times b \times \sqrt{2gh}\, dh$$

$$= C_d \times b \times \sqrt{2g} \int_{H_1}^{H_2} \sqrt{h}\, dh = C_d \times b \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_{H_1}^{H_2}$$

$$= \frac{2}{3} C_d \times b \sqrt{2g} \left[H_2^{3/2} - H_1^{3/2} \right].$$

FLOW THROUGH TOTALLY SUBMERGED ORIFICE

Fully sub-merged orifice is one which has its whole of the outlet side sub-merged under liquid so that it discharges a jet of liquid into the liquid of the same kind. It is also called totally drowned orifice. Fig. 7.8 shows the fully sub-merged orifice. Consider two points (1) and (2), point 1 being in the reservoir on the upstream side of the orifice and point 2 being at the vena-contracta as shown in Fig. 7.8.

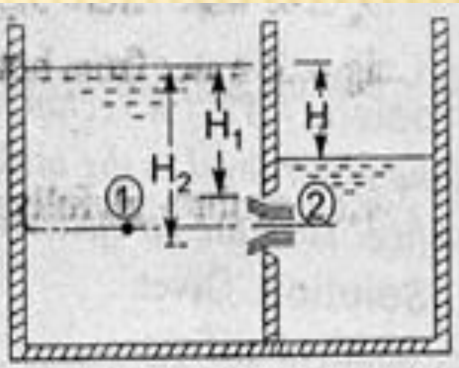


Fig. 7.8 *Fully sub-merged orifice.*

Let H_1 = Height of water above the top of the orifice on the upstream side.

H_2 = Height of water above the bottom of the orifice

H = Difference in water level

b = Width of orifice

C_d = Co-efficient of discharge.

FLOW THROUGH TOTALLY SUBMERGED ORIFICE

Height of water above the centre of orifice on upstream side

$$= H_1 + \frac{H_2 - H_1}{2} = \frac{H_1 + H_2}{2} \quad \text{...(1)}$$

Height of water above the centre of orifice on downstream side

$$= \frac{H_1 + H_2}{2} - H \quad \text{...(2)}$$

Applying Bernoulli's equation at (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} \quad [\because z_1 = z_2]$$

Now $\frac{p_1}{\rho g} = \frac{H_1 + H_2}{2}$, $\frac{p_2}{\rho g} = \frac{H_1 + H_2}{2} - H$ and V_1 is negligible

FLOW THROUGH TOTALLY SUBMERGED ORIFICE

$$\therefore \quad \frac{H_1 + H_2}{2} + 0 = \frac{H_1 + H_2}{2} - H + \frac{V_2^2}{2g}$$

$$\therefore \quad \frac{V_2^2}{2g} = H$$

$$\therefore \quad V_2 = \sqrt{2gH}$$

Area of orifice $= b \times (H_2 - H_1)$

$\therefore$ Discharge through orifice $= C_d \times \text{Area} \times \text{Velocity}$

$$= C_d \times b\,(H_2 - H_1) \times \sqrt{2gH}$$

$$\therefore \quad Q = C_d \times b\,(H_2 - H_1) \times \sqrt{2gH}\,.$$

FLOW THROUGH PARTIALLY SUBMERGED ORIFICE

Partially sub-merged orifice is one which has its outlet side partially sub-merged under liquid as shown in Fig. 7.9. It is also known as partially drowned orifice. Thus the partially sub-merged orifice has two portions. The upper portion behaves an orifice discharging free while the lower portion behaves as a sub-merged orifice. Only a large orifice can behave as a partially sub-merged orifice. The total discharge Q through partially sub-merged orifice is equal to the discharges through free and the sub-merged portions.

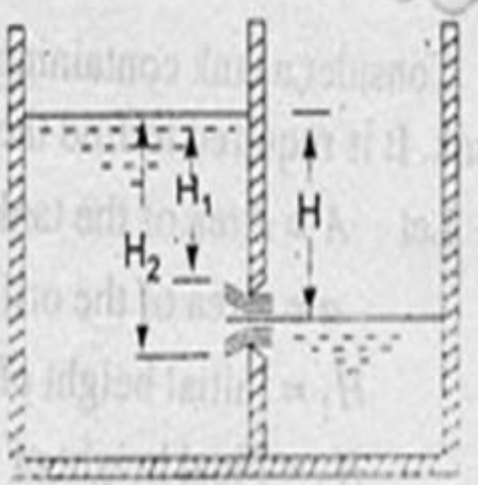


Fig. 7.9 *Partially sub-merged orifice.*

FLOW THROUGH PARTIALLY SUBMERGED ORIFICE

Discharge through the sub-merged portion is given by equation

$$Q_1 = C_d \times b \times (H_2 - H) \times \sqrt{2gH}$$

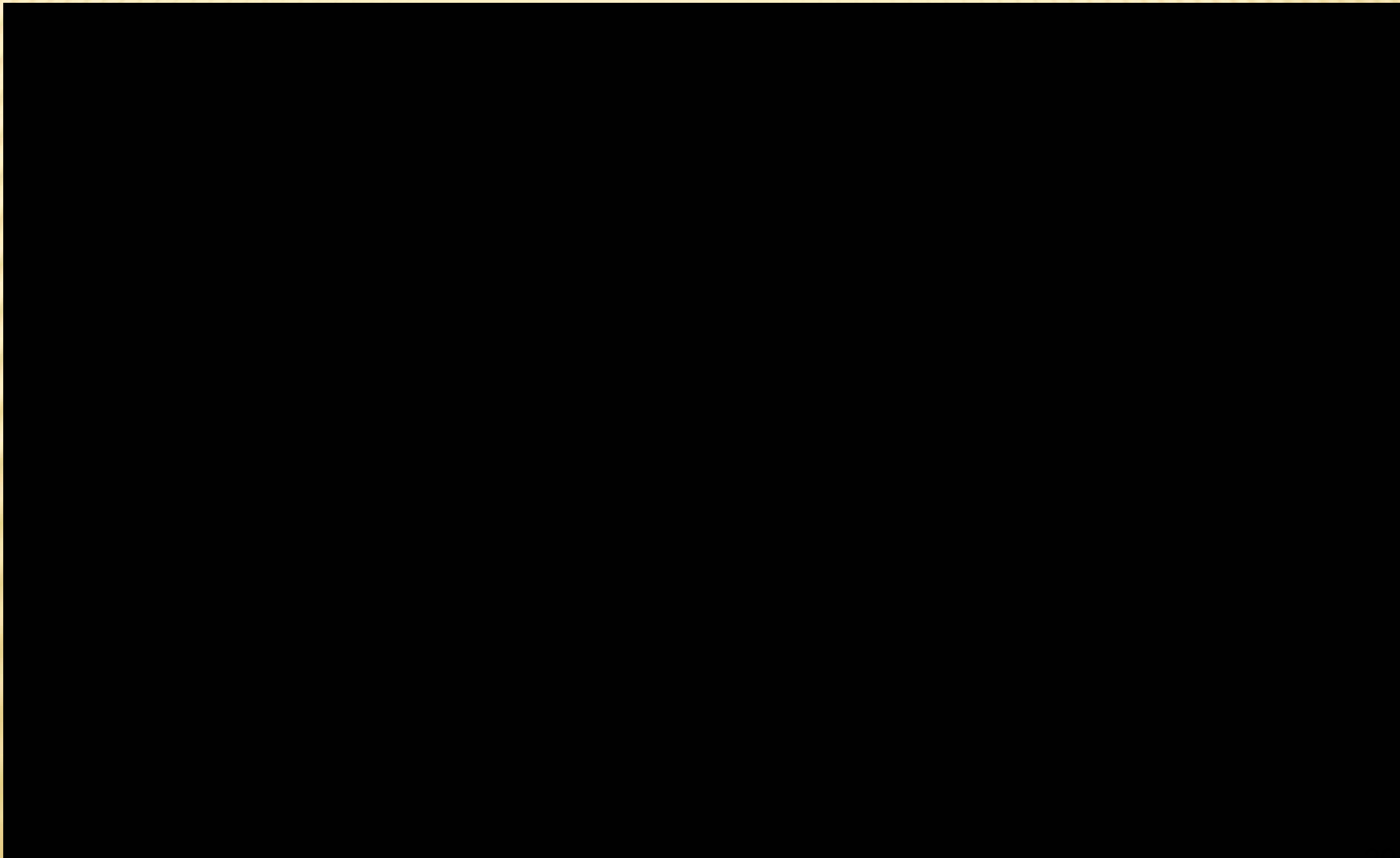
Discharge through the free portion is given by

$$Q_2 = \frac{2}{3} C_d \times b \times \sqrt{2g}\,[H^{3/2} - H_1^{3/2}]$$

$\therefore$ Total discharge $\quad Q = Q_1 + Q_2$

$$= C_d \times b \times (H_2 - H) \times \sqrt{2gH} + \frac{2}{3} C_d \times b \times \sqrt{2g}\,[H^{3/2} - H_1^{3/2}]$$

VIDEO!!



FLOW MEASUREMENT DEVICE : VENTURIMETER

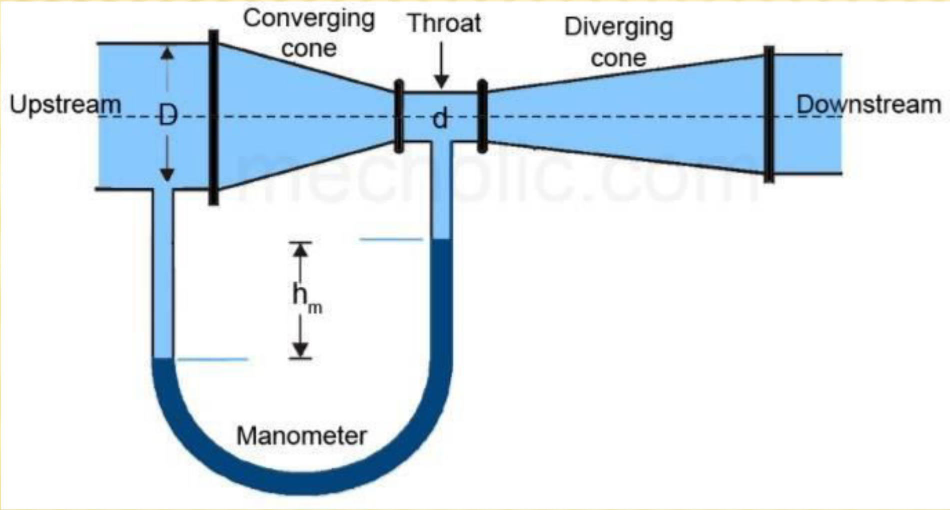
A venturimeter is a device used for measuring the rate of flow of fluid flowing throught the pipe.It consists of three parts.

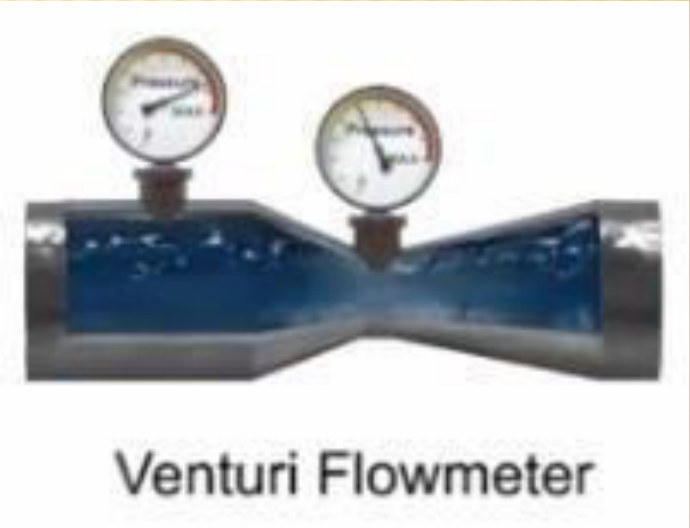
1.A short converging part

2.Throat

3.Diverging part

It is based on the principle of Bernoullis equation.





EXPRESSION FOR RATE OF FLOW THROUGH VENTURIMETER

Consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing (say water), as shown in Fig. 6.9.

Let d_1 = diameter at inlet or at section (1),

p_1 = pressure at section (1)

v_1 = velocity of fluid at section (1),

a = area at section (1) = $\frac{\pi}{4} d_1^2$

and d_2, p_2, v_2, a_2 are corresponding values at section (2).

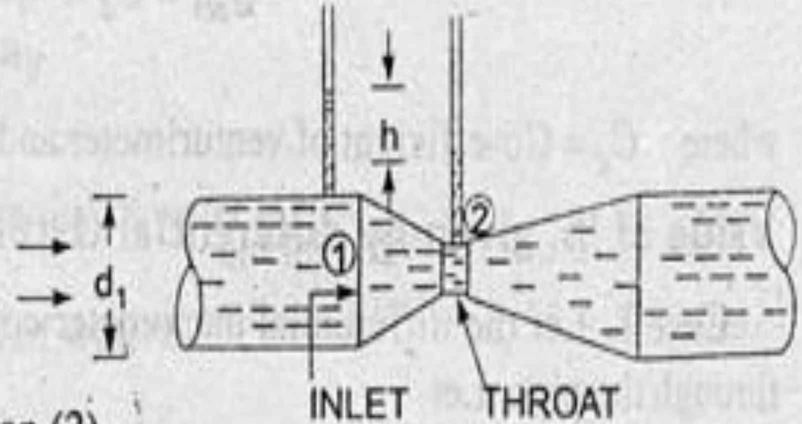


Fig. 6.9 *Venturimeter.*

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As pipe is horizontal, hence $z_1 = z_2$

$$\therefore \quad \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \quad \text{or} \quad \frac{p_1 - p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

EXPRESSION FOR RATE OF FLOW THROUGH VENTURIMETER

But $\frac{p_1 - p_2}{\rho g}$ is the difference of pressure heads at sections 1 and 2 and it is equal to h or $\frac{p_1 - p_2}{\rho g} = h$

Substituting this value of $\frac{p_1 - p_2}{\rho g}$ in the above equation, we get

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad \text{...(6.6)}$$

Now applying continuity equation at sections 1 and 2

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad v_1 = \frac{a_2 v_2}{a_1}$$

Substituting this value of v_1 in equation (6.6)

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right] = \frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

or

$$v_2^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$$

EXPRESSION FOR RATE OF FLOW THROUGH VENTURIMETER

$$\therefore \quad v_2 = \sqrt{2gh\frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}$$

$\therefore$ Discharge, $Q = a_2 v_2$

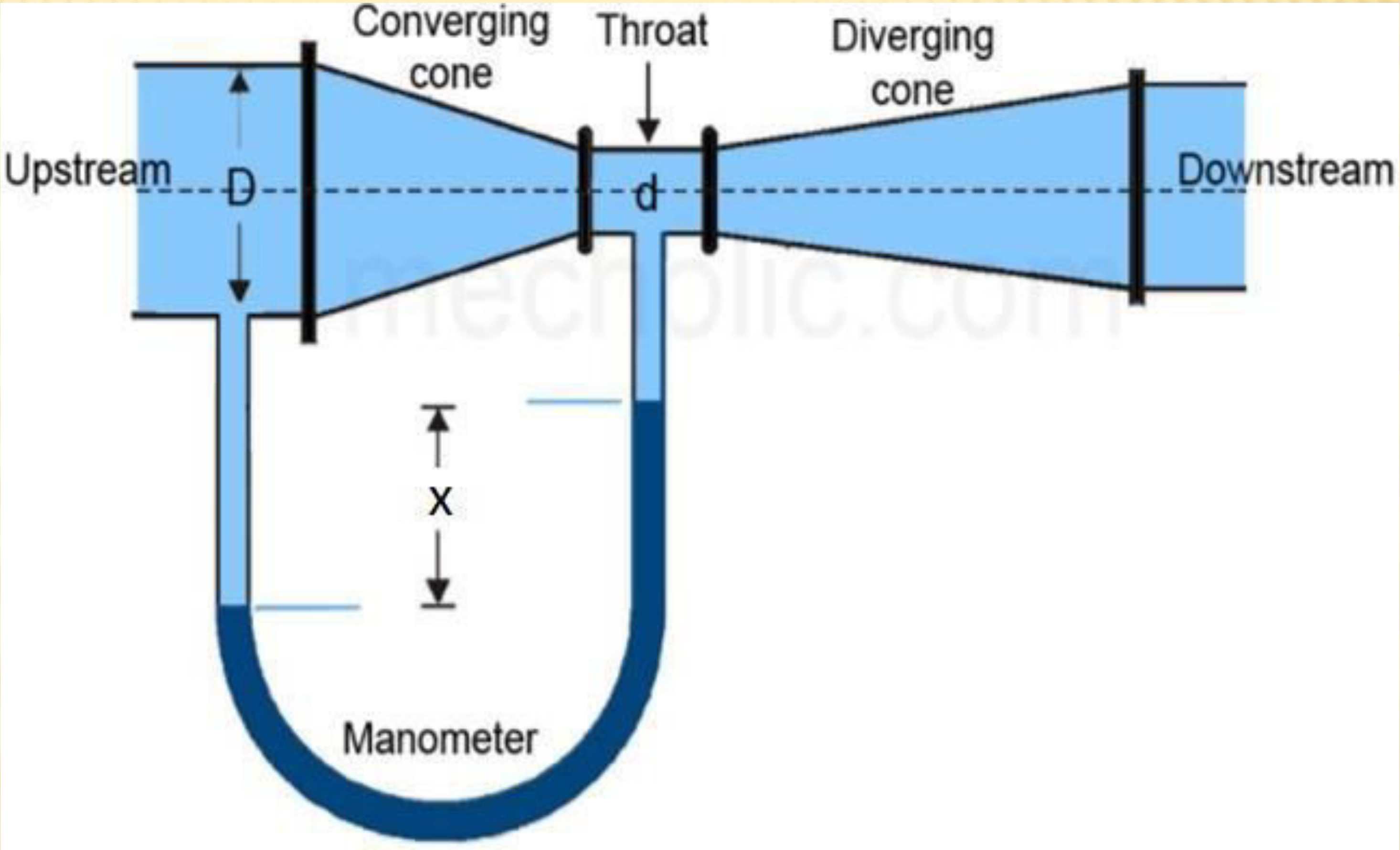
$$= a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad \text{...(6.7)}$$

Equation (6.7) gives the discharge under ideal conditions and is called, theoretical discharge. Actual discharge will be less than theoretical discharge.

$$\therefore \quad Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad \text{...(6.8)}$$

where C_d = Co-efficient of venturimeter and its value is less than 1.

MANOMETER CONNECTED TO VENTURI-METER



EXPRESSION FOR RATE OF FLOW THROUGH VENTURIMETER

Value of 'h' given by differential U-tube manometer

Case I. Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe. Let

S_h = Sp. gravity of the heavier liquid
S_o = Sp. gravity of the liquid flowing through pipe
x = Difference of the heavier liquid column in U-tube

Then
$$h = x\left[\frac{S_h}{S_o} - 1\right] \quad \text{...(6.9)}$$

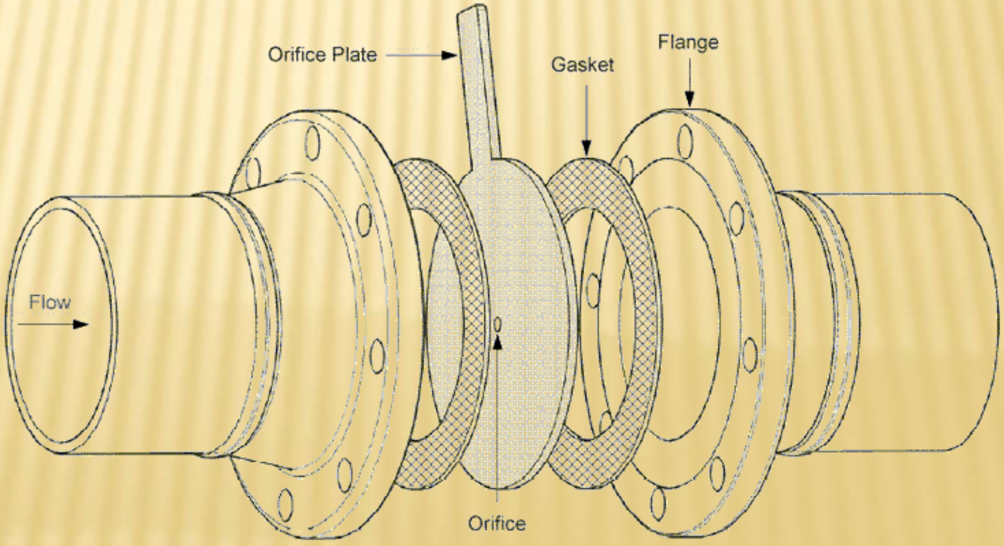
Case II. If the differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given by

$$h = x\left[1 - \frac{S_l}{S_o}\right] \quad \text{...(6.10)}$$

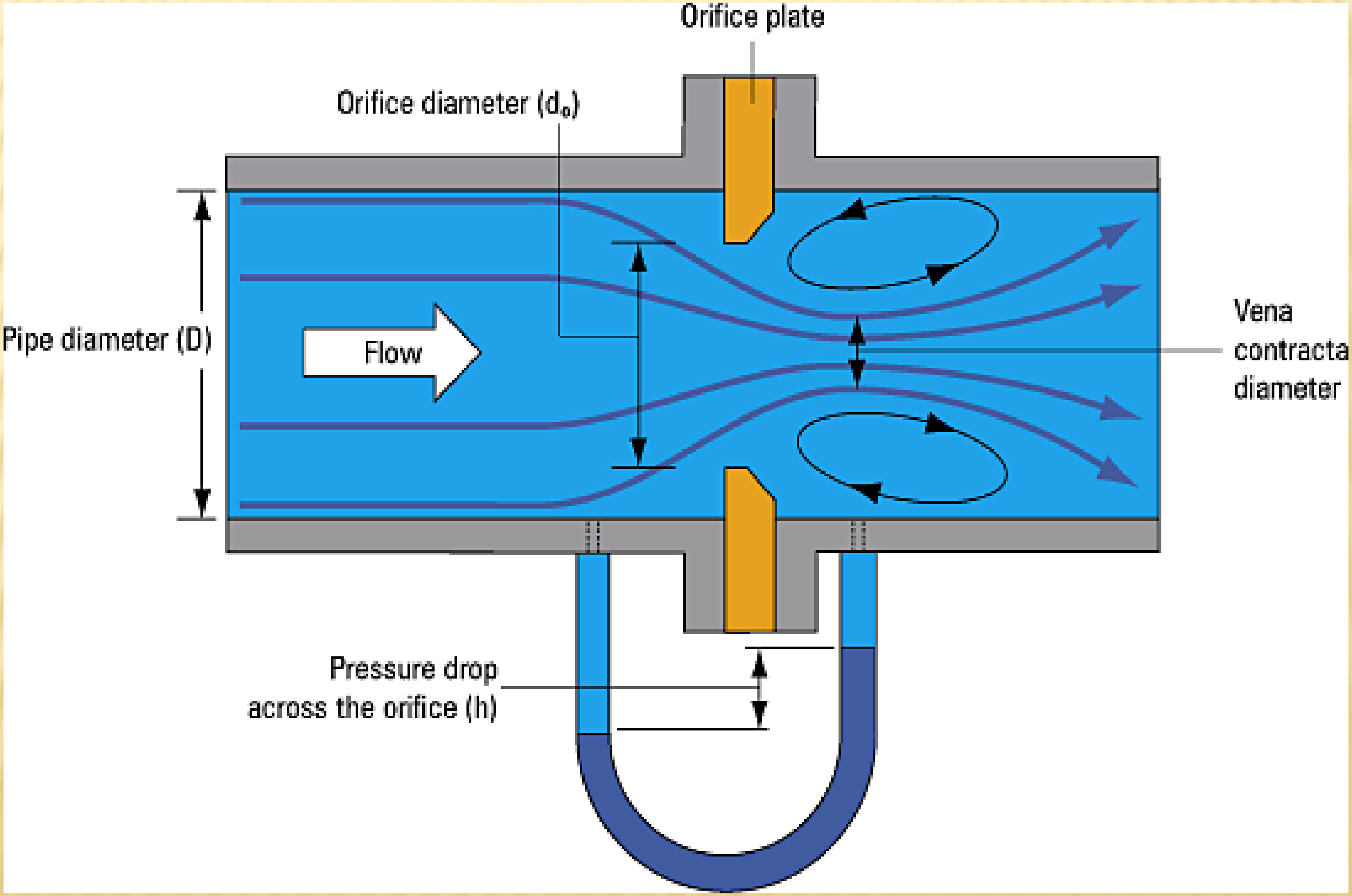
where S_1 = Sp. gr. of lighter liquid in U-tube
S_0 = Sp. gr. of fluid flowing through pipe
x = Difference of the lighter liquid columns in U-tube.

FLOW MEASUREMENT DEVICE: ORIFICE METER/ORIFICE PLATE

- It is a device used for measuring rate of flow of fluid through a pipe.
- It is cheaper in comparison to venturimeter but it works on same principle as venturimeter.
- It consists of a flat circular plate which has a circular sharp edged hole called orifice which is concentric with the pipe.
- The orifice diameter is generally kept 0.5 times the diameter of pipe.(normally 0.4-0.8 times pipe diameter can be used)
- A differential manometer is connected at a distance(1.5-2 times pipe dia)in u/s and (0.5 times orifice dia)distance in d/s from orifice plate.



FLOW MEASUREMENT DEVICE: ORIFICE METER/ORIFICE PLATE



EXPRESSION FOR RATE OF FLOW THROUGH ORIFICE PLATE/METER

Let p_1 = pressure at section (1),
v_1 = velocity at section (1),
a_1 = area of pipe at section (1), and

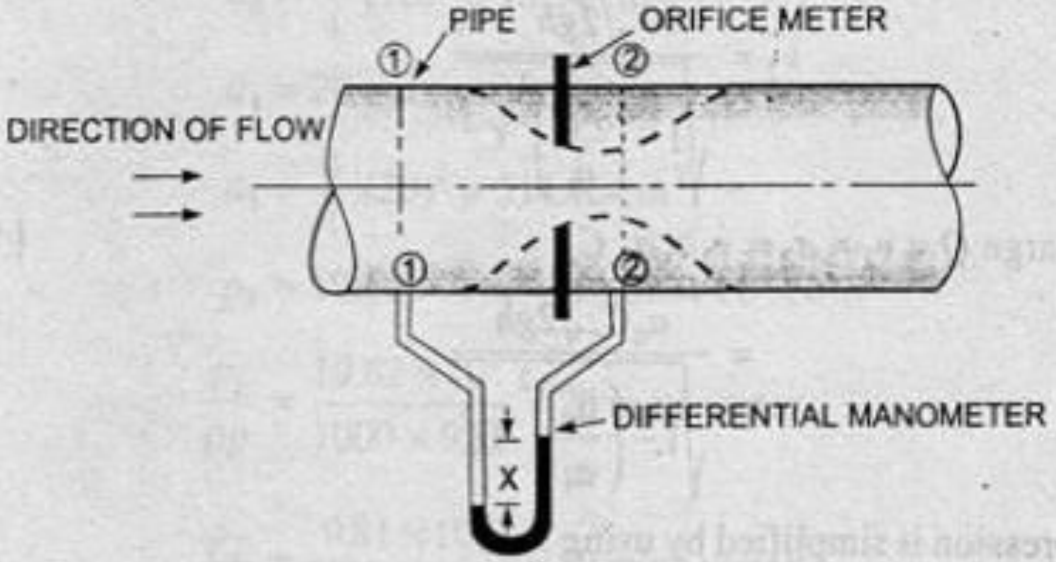


Fig. 6.12. *Orifice meter.*

p_2, v_2, a_2 are corresponding values at section (2). Applying Bernoulli's equation at sections (1) and (2), we get

EXPRESSION FOR RATE OF FLOW THROUGH ORIFICE PLATE/METER

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

or $$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But $$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = h = \text{Differential head}$$

$\therefore$ $$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad \text{or} \quad 2gh = {v_2}^2 - {v_1}^2$$

or $$v_2 = \sqrt{2gh + v_1^2} \qquad \ldots(i)$$

EXPRESSION FOR RATE OF FLOW THROUGH ORIFICE PLATE/METER

- For Orifice meter,

Q= $\frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$.

Where,

Cd= Coefficient of discharge for orifice meter

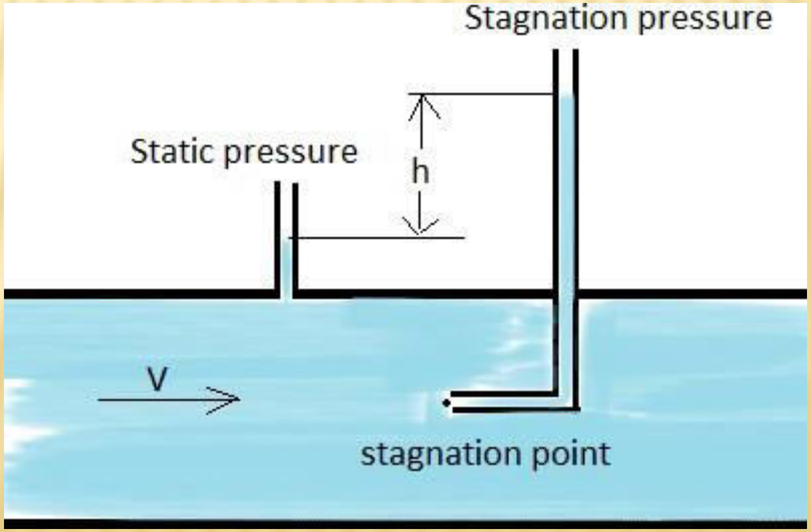
The Cd value of orifice meter is much smaller than that for venturimeter.

FLOW MEASUREMENT DEVICE: PITOT TUBE

- It is a device used for measuring the velocity of flow at any point in a pipe/channel.
- It is based on the principle that

"When velocity of flow at a point becomes zero, the pressure there is increased due to conversion of kinetic energy into pressure energy"

- The common type of pitot tube consists of a glass tube bent at right angles.



FLOW MEASUREMENT DEVICE: PITOT TUBE

- In fig, the lower end is bent through 90° upwards
- The liquid rises up in it due to conversion of kinetic energy into pressure energy.
- The velocity is measured by measuring the rise of liquid in the tube.

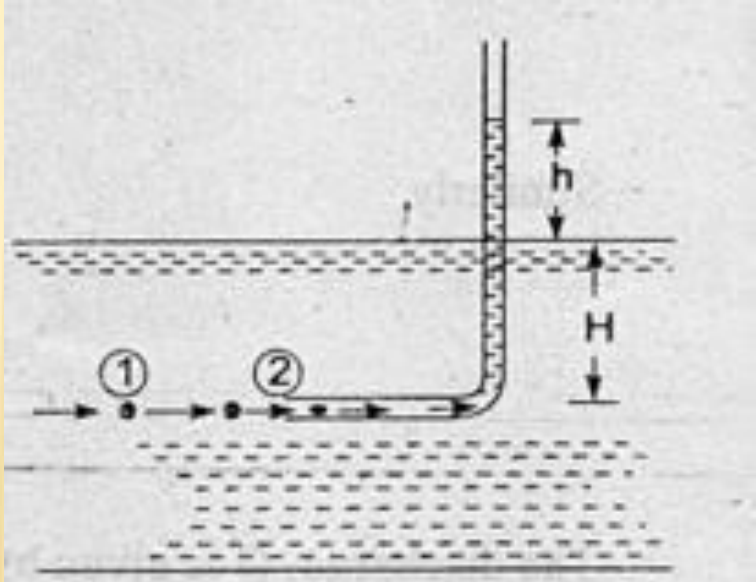


Fig. 6.13 *Pitot-tube.*

FLOW MEASUREMENT DEVICE: PITOT TUBE

Consider two points (1) and (2) at the same level in such a way that point (2) is just as the inlet of the pitot-tube and point (1) is far away from the tube.

Let p_1 = intensity of pressure at point (1)
v_1 = velocity of flow at (1)
p_2 = pressure at point (2)
v_2 = velocity at point (2), which is zero
H = depth of tube in the liquid
h = rise of liquid in the tube above the free surface.

Applying Bernoulli's equations at points (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_2^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

But $z_1 = z_2$ as points (1) and (2) are on the same line and $v_2 = 0$.

$$\frac{p_1}{\rho g} = \text{pressure head at (1)} = H$$

$$\frac{p_2}{\rho g} = \text{pressure head at (2)} = (h + H)$$

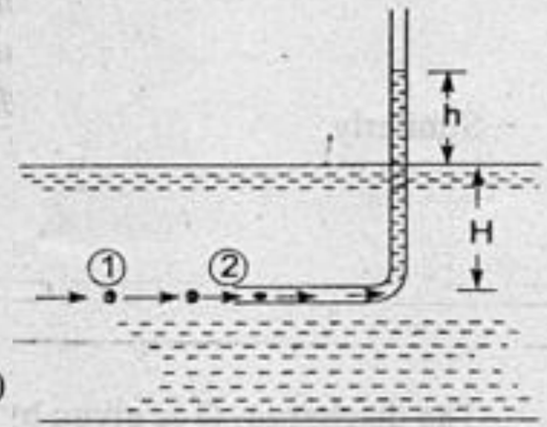


Fig. 6.13 *Pitot-tube.*

Substituting these values, we get

$$\therefore \quad H + \frac{v_1^2}{2g} = (h + H) \quad \therefore \quad h = \frac{v_1^2}{2g} \quad \text{or} \quad v_1 = \sqrt{2gh}$$

This is theoretical velocity. Actual velocity is given by

FLOW MEASUREMENT DEVICE: PITOT TUBE

$$(v_1)_{act} = C_v \sqrt{2gh}$$

where C_v = Co-efficient of pitot-tube

$\therefore$ Velocity at any point $\quad v = C_v \sqrt{2gh} \quad$ ...(6.14)

Velocity of flow in a pipe by pitot-tube. For finding the velocity at any point in a pipe by pitot-tube, the following arrangements are adopted :

1. Pitot-tube along with a vertical piezometer tube as shown in Fig. 6.14.
2. Pitot-tube connected with piezometer tube as shown in Fig. 6.15.
3. Pitot-tube and vertical piezometer tube connected with a differential U-tube manometer as shown in Fig. 6.16.

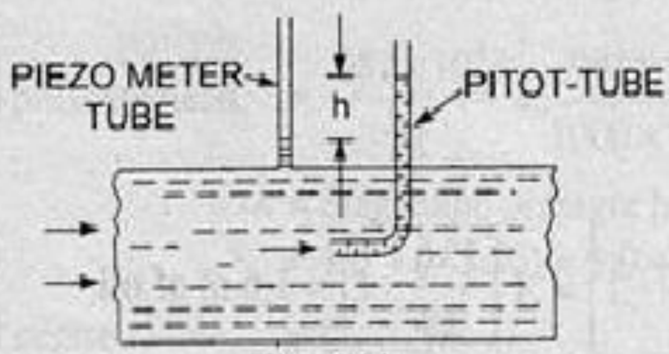


Fig. 6.14

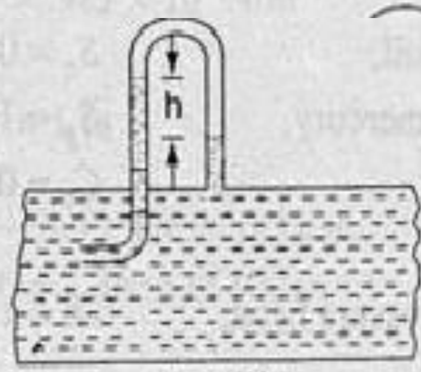


Fig. 6.15

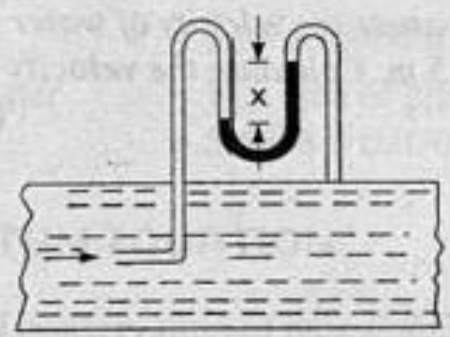
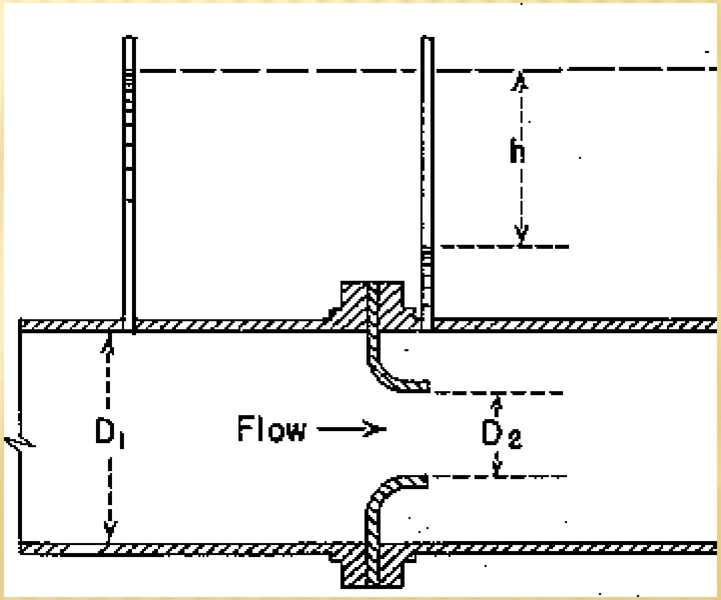


Fig. 6.16

FLOW MEASUREMENT DEVICE: NOZZLE METER

- It is used to measure discharge through the pipes.
- Nozzle meter is similar to venturimeter with its divergent part omitted so the basic equations are same.
- Its coefficient of discharge is same as that of venturimeter.

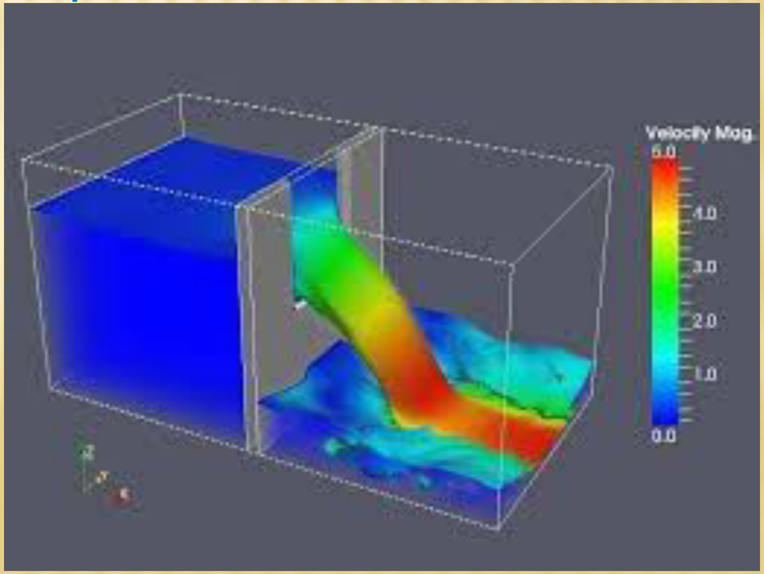


NOTCHES AND WEIRS

Notch

- A notch is a device used for measuring the rate of flow of liquid through a small channel or a tank.
- It may be defined as a opening in the side of the tank or small channel in such a way that the liquid surface in the tank/channel is below top edge of the opening.
- Notch is generally made of metallic plate.

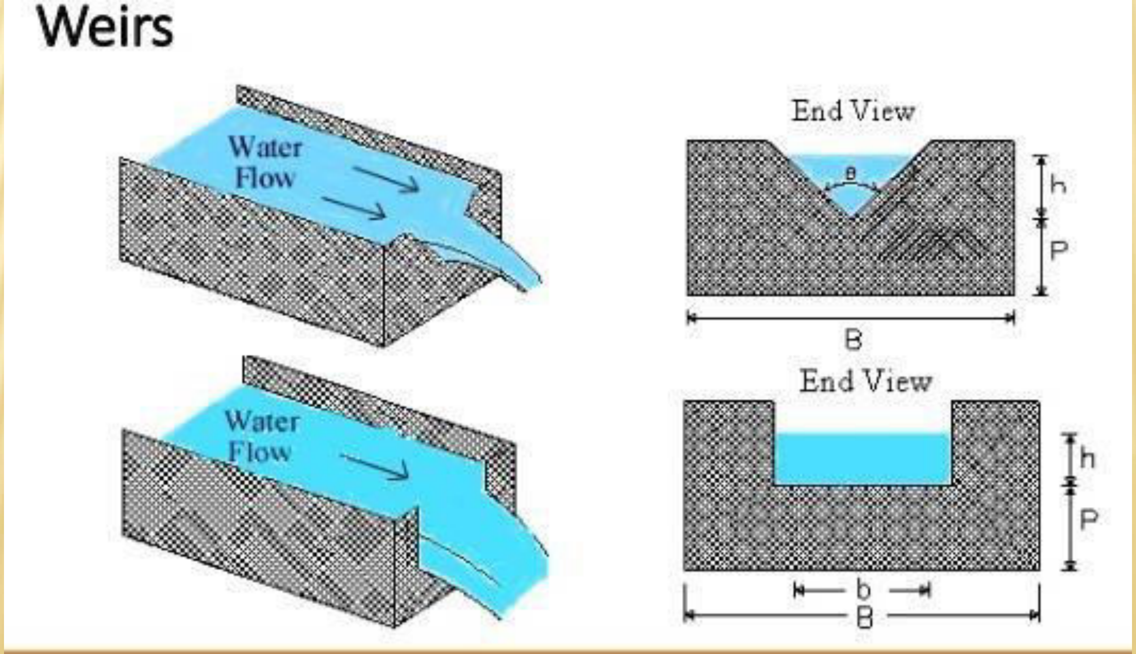
V Notch Weir



NOTCHES AND WEIRS

Weir

- A Weir is a concrete/ masonry structure placed in an open channel over which flow occurs.
- It is generally in the form of vertical wall, with the sharp edge at the top, running all the way across the open channel.
- Weir is of big size in comparison to notch.



Classification of notches and weirs

The notches are classified as :

1. According to the shape of the opening :
   (*a*) Rectangular notch,
   (*b*) Triangular notch,
   (*c*) Trapezoidal notch, and
   (*d*) Stepped notch.
2. According to the effect of the sides on the nappe :
   (*a*) Notch with end contraction.
   (*b*) Notch without end contraction or suppressed notch.

NOTCHES AND WEIRS

Classification of notches and weirs

Weirs are classified according to the shape of the opening the shape of the crest, the effect of the sides on the nappe and nature of discharge. The following are important classifications.

(*a*) According to the shape of the opening :

(*i*) Rectangular weir,
(*ii*) Triangular weir, and
(*iii*) Trapezoidal weir (Cippoletti weir)

(*b*) According to the shape of the crest :

(*i*) Sharp-crested weir,
(*ii*) Broad-crested weir,
(*iii*) Narrow-crested weir, and
(*iv*) Ogee-shaped weir.

(*c*) According to the effect of sides on the emerging nappe :

(*i*) Weir with end contraction, and
(*ii*) Weir without end contraction.

NOTCHES AND WEIRS

DISCHARGE OVER RECTANGULAR NOTCHES AND WEIRS

The expression for discharge over a rectangular notch or weir is the same.

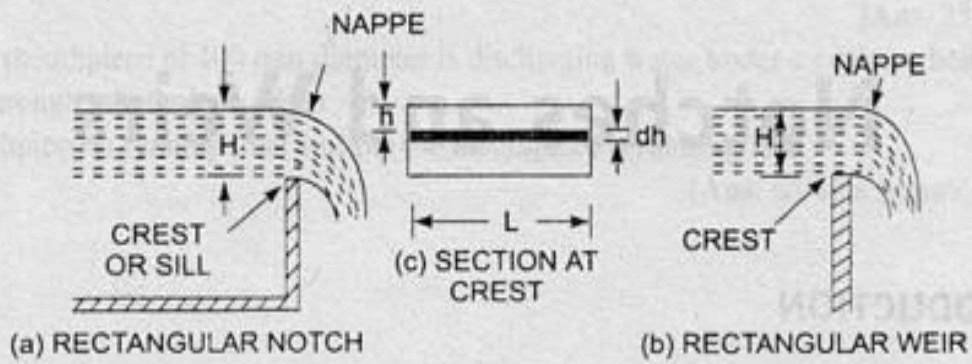


Fig. 8.1 *Rectangular notch and weir.*

Consider a rectangular notch or weir provided in a channel carrying water as shown in Fig. 8.1.

Let H = Head of water over the crest

L = Length of the notch or weir

Some terms:

Nappe/Vein: The sheet of water flowing through the notch or weir

Crest/Sill: The bottom edge of notch or top of weir over which water flows

Discharge over rectangular notches and weirs

For finding the discharge of water flowing over the weir or notch, consider an elementary horizontal strip of water of thickness dh and length L at a depth h form the free surface of water as shown in Fig. 8.1(c).

The area of strip $= L \times dh$

and theoretical velocity of water flowing through strip $= \sqrt{2gh}$

The discharge dQ, through strip is

$$dQ = C_d \times \text{Area of strip} \times \text{Theoretical velocity}$$

$$= C_d \times L \times dh \times \sqrt{2gh} \quad \ldots(i)$$

where C_d = Co-efficient of discharge.

The total discharge, Q, for the whole notch or weir is determined by integrating equation (i) between the limits 0 and H.

$$\therefore \quad Q = \int_0^H C_d \,.\, L \,.\, \sqrt{2gh} \,.\, dh = C_d \times L \times \sqrt{2g} \int_0^H h^{1/2}\, dh$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{1/2+1}}{\frac{1}{2}+1} \right]_0^H = C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H$$

$$= \frac{2}{3} C_d \times L \times \sqrt{2g}\, [H]^{3/2}. \quad \ldots(8.1)$$

VELOCITY OF APPROACH

It is the velocity with which the water approaches or reaches the weir/notch before it flows over it.

•If Va =velocity of approach

Then,

ha=additional head equal to $Va^2/2g$ due to velocity of approach acting on water flowing over the notch.

H+ha=initial height over the notch

ha= final height over the notch

VELOCITY OF APPROACH

Process of determining Va:

•Find discharge 'Q' over notch/weir neglecting the Va.

•Divide Q by cross-sectional area of channel on the u/s side of weir/notch to find Va.

$$V_a = \frac{Q}{\text{Area of channel}}$$

•Find additional head ha.

$$\left(h_a = \frac{V_a^2}{2g}\right).$$

•Calculate discharge again including Va.

Discharge over a rectangular weir, with velocity of approach

$$= \frac{2}{3} C_d \times L \times \sqrt{2g}\ [(H + h_a)^{3/2} - h_a^{3/2}]$$

Discharge over triangular notches and weirs

The expression for the discharge over a triangular notch or weir is the same. It is derived as :

Let H = head of water above the V- notch

θ = angle of notch

Consider a horizontal strip of water of thickness 'dh' at a depth of h from the free surface of water as shown in Fig. 8.3.

From Fig. 8.3 (b), we have

$$\tan\frac{\theta}{2} = \frac{AC}{OC} = \frac{AC}{(H-h)}$$

$$\therefore \quad AC = (H-h)\tan\frac{\theta}{2}$$

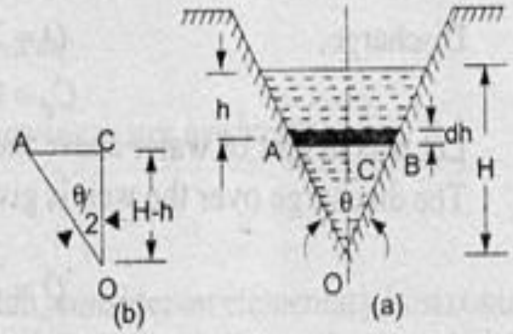


Fig. 8.3 *The triangular notch.*

Width of strip $= AB = 2AC = 2\,(\mathrm{H}-h)\tan\dfrac{\theta}{2}$

$\therefore$ Area of strip $= 2\,(\mathrm{H}-h)\tan\dfrac{\theta}{2}\times dh$

Discharge over triangular notches and weirs

The theoretical velocity of water through strip = $\sqrt{2gh}$

$\therefore$ Discharge, dQ, through the strip is

$$dQ = C_d \times \text{Area of strip} \times \text{Velocity (theoretical)}$$

$$= C_d \times 2(H-h)\tan\frac{\theta}{2} \times dh \times \sqrt{2gh}$$

$$= 2C_d (H-h)\tan\frac{\theta}{2} \times \sqrt{2gh} \times dh$$

$\therefore$ Total discharge, Q is

$$Q = \int_0^H 2C_d (H-h)\tan\frac{\theta}{2} \times \sqrt{2gh} \times dh$$

$$= 2C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\int_0^H (H-h)h^{1/2}\,dh$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\int_0^H (Hh^{1/2} - h^{3/2})\,dh$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{Hh^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\right]_0^H$$

Discharge over triangular notches and weirs

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{2}{3} H.H^{3/2} - \frac{2}{5} H^{5/2}\right]$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{2}{3} H^{5/2} - \frac{2}{5} H^{5/2}\right]$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{4}{15} H^{5/2}\right]$$

$$= \frac{8}{15} C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2} \qquad \text{...(8.2)}$$

For a right-angled *V*-notch, if $C_d = 0.6$

$$\theta = 90°, \quad \therefore \quad \tan\frac{\theta}{2} = 1$$

Discharge

$$Q = \frac{8}{15} \times 0.6 \times 1 \times \sqrt{2 \times 9.81} \times H^{5/2} \qquad \text{...(8.3)}$$

$$= 1.417\, H^{5/2}.$$

NO

NOTCHES AND WEIRS

Discharge over triangular notches and weirs

If Va is taken into account,
Then ha=additional head of water flowing over the weir/notch
Or ha = $Va^2/2g$

The discharge Q over triangular notch/weir may be modified as,

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2}\left[(H + h_a)^{5/2} - h_a^{5/2}\right]$$

NOTCHES AND WEIRS

Advantages of triangular notches/weirs over rectangular notch/weir

Triangular notch/weir is preferred because:

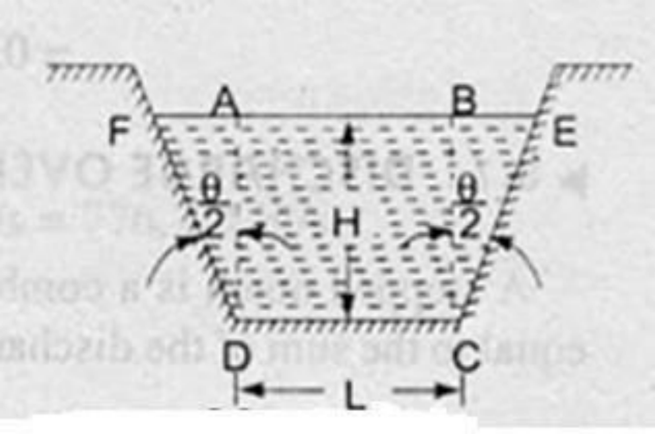
1. The expression for discharge for a right angled V-notch/weir is very simple.
2. Triangular notch gives accurate results while measuring low discharge in comparison to rectangular notch/weir.
3. Only one reading of H is required for computation of discharge in case of triangular notch.
4. Ventilation of triangular notch is not necessary.

NOTCHES AND WEIRS

Discharge over trapezoidal notches and weirs

A trapezoidal notch/weir is a combination of rectangular and triangular notch/weir.

Thus the total discharge =discharge through a rectangular weir/notch + discharge through a triangular notch/weir.



Let,

H=height of water over the notch.

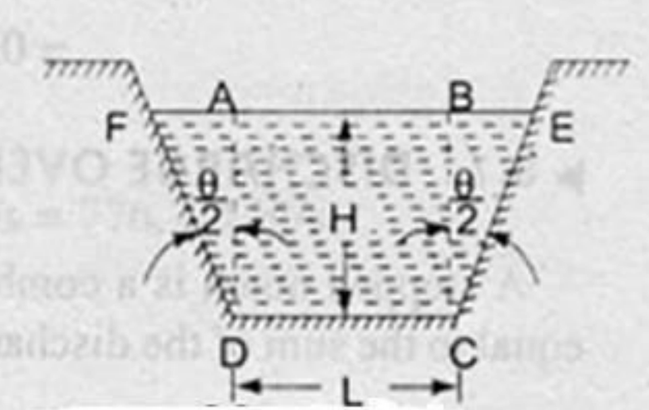
L=length of the crest of the notch.

C_{d1}=Coefficient of discharge through rectangular portion ABCD

C_{d2}=coefficient of discharge for triangular portion(FAD and BCE)

NOTCHES AND WEIRS

DISCHARGE OVER TRAPEZOIDAL NOTCHES AND WEIRS



The discharge through rectangular portion $ABCD$ is given by (8.1)

or
$$Q_1 = \frac{2}{3} \times C_{d_1} \times L \times \sqrt{2g} \times H^{3/2}$$

The discharge through two triangular notches FDA and BCE is equal to the discharge through a single triangular notch of angle θ and it is given by equation (8.2) as

$$Q_2 = \frac{8}{15} \times C_{d_2} \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

∴ Discharge through trapezoidal notch or weir $FDCEF = Q_1 + Q_2$

$$= \frac{2}{3} C_{d_1} L \sqrt{2g} \times H^{3/2} + \frac{8}{15} C_{d_2} \times \tan\theta/2 \times \sqrt{2g} \times H^{5/2}. \quad ...(8.4)$$

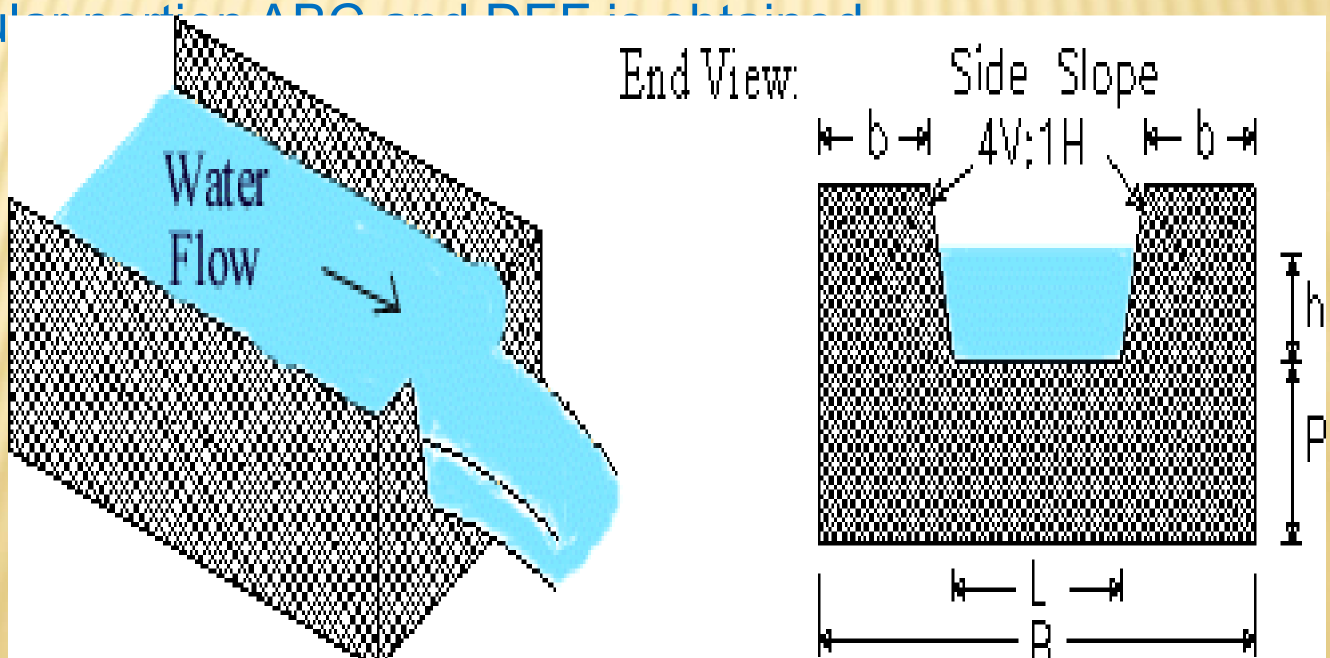
NOTCHES AND WEIRS

- Discharge over Cippoletti notches and weirs

Cipolletti weir $\left(i.e., \frac{\theta}{2} = 14^\circ\right)$ al weir having side slopes of 1 horizontal to 4 vertical

By providing slopes on sides , an increase in discharge through the

- triangular portion ABC and DEF is obtained.



NOTCHES AND WEIRS

DISCHARGE OVER STEPPED NOTCHES AND WEIRS

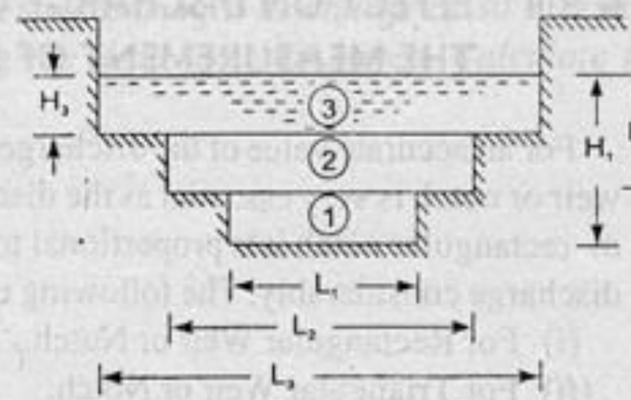
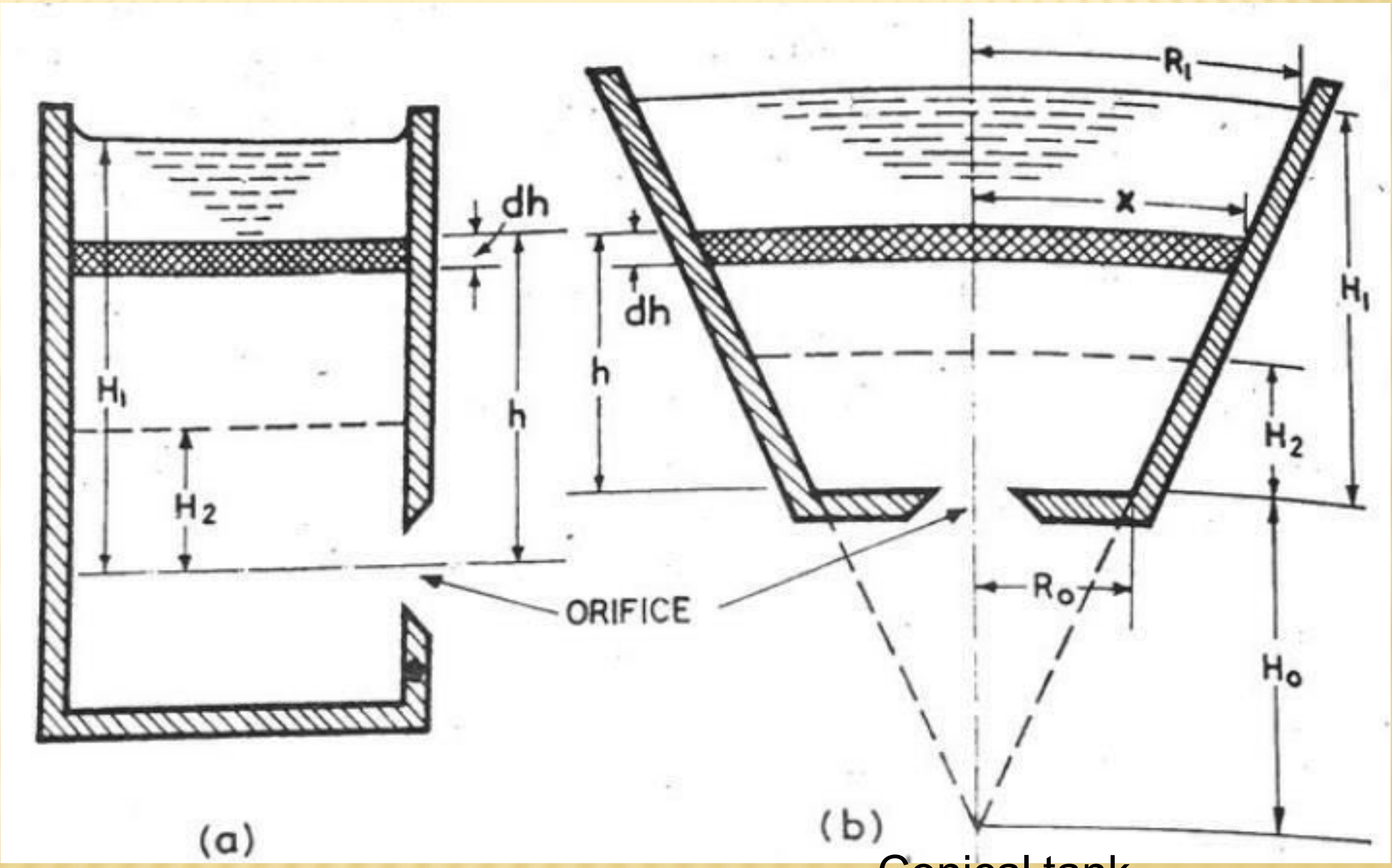


Fig. 8.6 *The stepped notch.*

$\therefore$ Total discharge $Q = Q_1 + Q_2 + Q_3$

or
$$Q = \frac{2}{3} \times C_d \times L_1 \times \sqrt{2g}\,[H_1^{3/2} - H_2^{3/2}]$$
$$+ \frac{2}{3} C_d \times L_2 \times \sqrt{2g}\,[H_2^{3/2} - H_3^{3/2}] + \frac{2}{3} C_d \times L_3 \times \sqrt{2g} \times H_3^{3/2}.$$

EMPTYING AND FILLING OF RESERVOIR WITHOUT INFLOW : RECTANGULAR/CYLINDRICAL TANK



Cylindrical/Rectangular tank

Conical tank

EMPTYING AND FILLING OF RESERVOIR WITHOUT INFLOW:RECTANGULAR TANK

Consider,

H_1=height of liquid above the center of the orifice provided on side/bottom of the tank

t= time required for liquid to fall from H1 to H2 above the center of opening

Let at any instant ,

h= height of liquid above orifice at an instant

Let the liquid surface fall by small amount dh in time dt.

A=horizontal cross-sectional of the tank

Q= discharge through the orifice

Volume of liquid discharged during time dt is Qdt.

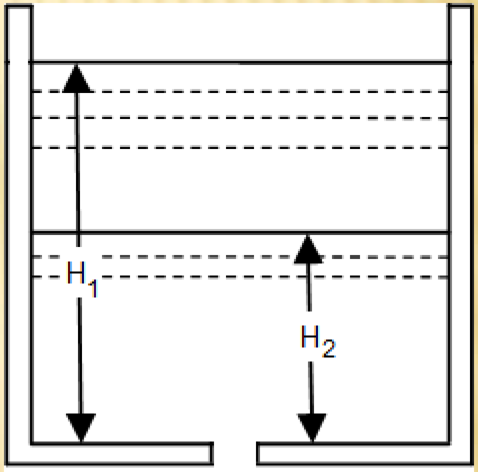


Fig : Tank with an orifice at its bottom

EMPTYING AND FILLING OF RESERVOIR WITHOUT INFLOW:RECTANGULAR TANK

As the volume of liquid leaving the tank is equal to the volume of liquid flowing through the orifice/mouthpiece during the same interval of time. we have,

A(-dh)=Qdt (negative sign because as time increases head decreases)

If a=cross-sectional area of orifice/mouthpiece and

Cd=coefficient of discharge then,

$$Q = C_d a \sqrt{2gh}$$

and by substitution

$$-Adh = C_d a (\sqrt{2gh})dt$$

or

$$dt = -\frac{Adh}{C_d a \sqrt{2gh}}$$

EMPTYING AND FILLING OF RESERVOIR WITHOUT INFLOW :RECTANGULAR TANK

By integrating both the sides of the above expression, we get

$$\int_0^t dt = -\int_{H_1}^{H_2} \frac{A\,dh}{C_d a\sqrt{2gh}}$$

or

$$t = -\int_{H_1}^{H_2} \frac{A\,dh}{C_d a\sqrt{2gh}} \qquad \text{...(9.39)}$$

Equation 9.39 may be evaluated if the shape of the tank is known. The tanks of the following shapes are commonly found in practice.

(*i*) Cylindrical (or rectangular or prismatic, with constant horizontal cross-sectional area).

(*ii*) Conical.

(*iii*) Hemispherical.

EMPTYING AND FILLING OF RESERVOIR WITHOUT INFLOW:RECTANGULAR TANK

(i) Cylindrical (or rectangular or prismatic, with constant horizontal cross-sectional area).

$$t = -\frac{A}{C_d a\sqrt{2g}} \int_{H_1}^{H_2} h^{-1/2}\, dh$$

$$t = \frac{2A}{C_d a\sqrt{2g}} (H_1^{1/2} - H_2^{1/2})$$

If the tank is completely emptied then $H_2 = 0$ becomes

$$t = \frac{2A\,(H_1^{1/2})}{C_d a\sqrt{2g}}$$

EMPTYING AND FILLING OF RESERVOIR WITHOUT INFLOW: CONICAL TANK

(*ii*) **Conical tank.** Generally a conical tank has a shape of a frustrum of a cone. In this case the horizontal cross-sectional area A varies. Thus as shown in Fig. 9.16 (*b*)

$$A = \pi x^2$$

where x is the radius of the cone at a height h above the bottom. From the similar triangles, we have

$$\frac{R_1}{(H_1 + H_0)} = \frac{x}{(h + H_0)}$$

or

$$x = \frac{R_1(h + H_0)}{(H_1 + H_0)}$$

Then from equation 9.39

$$t = -\frac{\pi R_1^2}{C_d a\sqrt{2g}(H_1 + H_0)^2} \int_{H_1}^{H_2} \frac{(H_0 + h)^2}{\sqrt{h}} dh$$

or

$$t = \frac{\pi R_1^2}{C_d a\sqrt{2g}(H_1 + H_0)^2} \times \left[\frac{2}{5} h^{5/2} + 2H_0^2 h^{1/2} + \frac{4}{3} H_0 h^{3/2}\right]_{H_1}^{H_2} \quad \ldots(9.43)$$

EMPTYING AND FILLING OF RESERVOIR WITHOUT INFLOW:CONICAL TANK

In the above expression the value of H_0 may be obtained if the radius R_0 at the bottom of the vessel is known. Thus again by similar triangles

$$\frac{R_1}{H_1 + H_0} = \frac{R_0}{H_0}$$

or

$$H_0 = \frac{R_0 H_1}{(R_1 - R_0)}$$

(*iii*) **Hemispherical tank.** In this case too the horizontal cross-sectional area is varying. As shown in Fig. 9.16 (*d*)

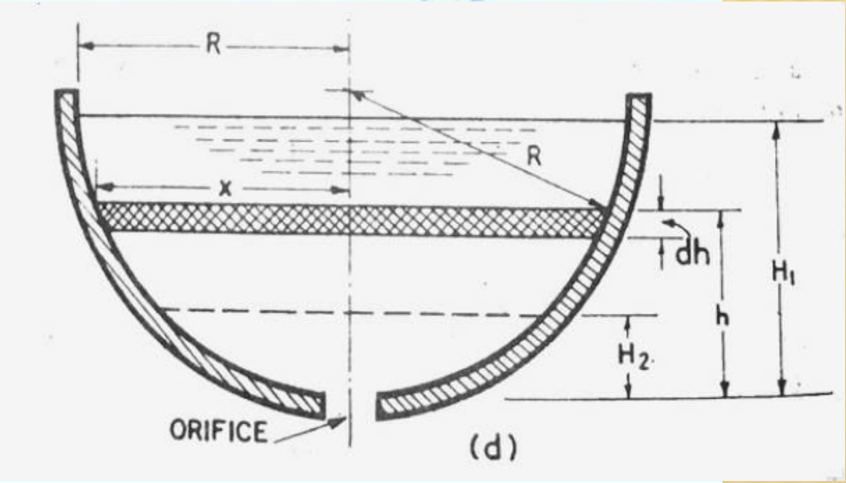
$$A = \pi x^2$$

and

$$x = \sqrt{2Rh - h^2}$$

where R is the radius of tank. Then from equation 9.39

$$t = -\frac{\pi}{C_d a\sqrt{2g}} \int_{H_1}^{H_2} \frac{(2Rh - h^2)}{\sqrt{h}} dh$$



EMPTYING AND FILLING OF RESERVOIR WITHOUT INFLOW:CONICAL TANK

or

$$t = \frac{2\pi}{C_d a\sqrt{2g}} \times \left[\frac{2}{3}R(H_1^{3/2} - H_2^{3/2}) - \frac{1}{5}(H_1^{5/2} - H_2^{5/2})\right]$$

Now if the tank (or vessel) was full at the beginning and it is completely emptied, then

$$H_1 = R$$

and

$$H_2 = 0$$

Equation 9.44 then becomes

$$t = \frac{14\pi R^{5/2}}{15 C_d a\sqrt{2g}}$$

EMPTYING AND FILLING OF RESERVOIR WITH INFLOW: RECTANGULAR TANK

Consider,

A=cross-sectional area of tank
a= cross-sectional area of orifice/mouthpiece
Q=constant inflow of liquid which is also discharging through orifice
t= time in which liquid level changes from H_1 to H_2 above the center of the orifice
dt = time required to increase the water level by dh.
q= discharge through the orifice

Volume of liquid added to the tank=Adh
In time dt the volume of inflow in tank=Qdt
Volume of outflow through the orifice=qdt

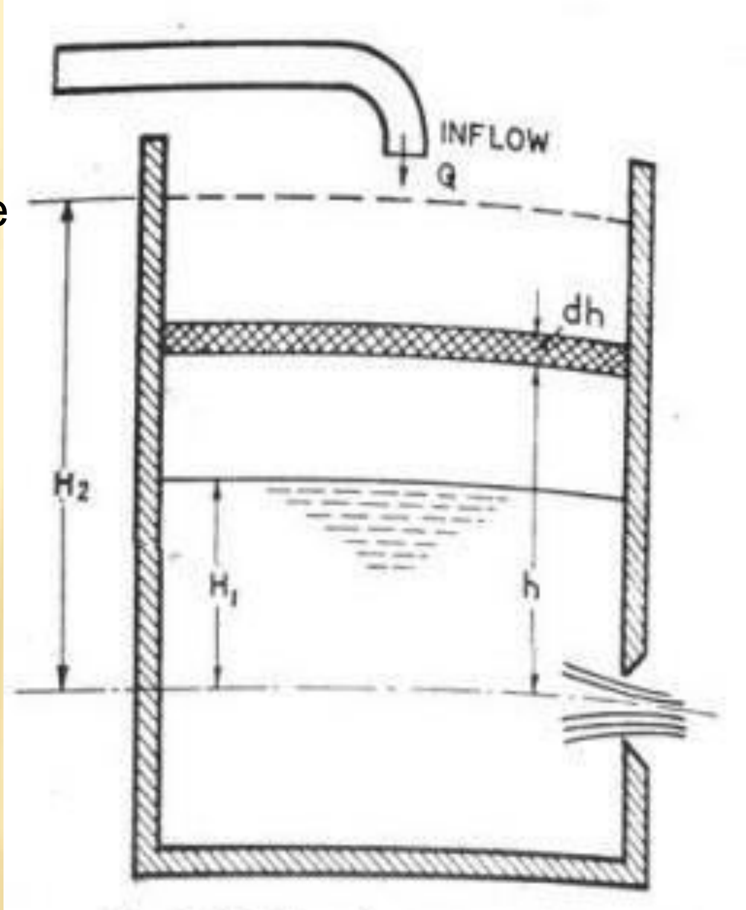


Fig. 9.17. Flow from a vessel with inflow

EMPTYING AND FILLING OF RESERVOIR WITH INFLOW: RECTANGULAR TANK

Since $q = C_d a \sqrt{2gh}$

$= K\sqrt{h}$

where $K = C_d a \sqrt{2g}$

$$qdt = K\sqrt{h}\ dt$$

Thus net volume of liquid added to the tank during time dt is

$$(Qdt - qdt) = (Q - K\sqrt{h})\ dt$$

Thus equating the two, we have

$$Adh = (Q - K\sqrt{h})\ dt$$

or
$$dt = \frac{Adh}{Q - K\sqrt{h}}$$

By integrating this equation the time required to raise the liquid surface from the height H_1 to H_2 may be obtained. Thus

EMPTYING AND FILLING OF RESERVOIR WITH INFLOW: RECTANGULAR TANK

$$\int_0^t dt = \int_{H_1}^{H_2} \frac{A\,dh}{Q - K\sqrt{h}}$$

or

$$t = \int_{H_1}^{H_2} \frac{A\,dh}{Q - K\sqrt{h}} \qquad \ldots(i)$$

Let $Q - K\sqrt{h} = z$

or

$$h = \frac{(Q-z)^2}{K^2}$$

Differentiating with respect to z

$$dh = -\frac{2(Q-z)}{K^2}\,dz$$

EMPTYING AND FILLING OF RESERVOIR WITH INFLOW: RECTANGULAR TANK

Substituting this value of dh and h in equation (i), we have

$$t = -\frac{2A}{K^2}\int\left(\frac{Q-z}{z}\right)dz$$

or $$t = -\frac{2A}{K^2}\left[Q\log_e z - z\right]$$

or $$t = -\frac{2A}{K^2}\left[Q\log_e\left(Q-K\sqrt{h}\right)-\left(Q-K\sqrt{h}\right)\right]_{H_1}^{H_2}$$

or $$t = -\frac{2A}{K^2}\left[Q\log_e\left(\frac{Q-K\sqrt{H_2}}{Q-K\sqrt{H_1}}\right)+K\left(\sqrt{H_2}-\sqrt{H_1}\right)\right] \quad ...(9.46)$$

Equation 9.46 can also be used to compute the time required to lower the liquid surface from the initial height H_1 to another height H_2, in which case equation 9.46 gives negative result.